	Building a Stereo-angle into strip-sensors for the ATLAS-Upgrade Inner-Tracker Endcaps		
ATLAS Project Doc. No: **None**	*Institute Doc. No:* **petalwafer**	*Created:* **06/01/12** *Modified:* **15/08/12**	*Page:* **1 of 14** *Rev:* **G**

Note

Building a Stereo-angle into strip-sensors for the ATLAS-Upgrade Inner-Tracker Endcaps

The Strips Endcap detector for the ATLAS Upgrade needs several sensor shapes, each of which is approximately a wedge shape like the current SCT. For the Endcap to use a stave-like approach as proposed for the barrel, care is needed to design the shapes to avoid clashes and minimise gaps between them. This note gives the basic formulae for one way of building up a petal. It allows for a stereo-angle to be built into the wafer, and takes into account the maximum usable wafer size.

Prepared by:	*Checked by:*	*Approved by:*
N.P. Hessey	**Eric Hennes**	**Nobody**

Distribution List:
Public

History of Changes

Rev. No.	*Date*	*Pages*	*Made by*	*Description of changes*
A	06/01/2010	All	N.P. Hessey	Original document
B	13/01/2010	All	N.P. Hessey	Corrections from Eric. Reverse y. Call angles ϕ.
C	01/02/2012	9 & ff	N.P. Hessey	Add guard ring/outer dimensions
D	03/04/2012	10 & ff	N.P. Hessey	Calculate strip end points
E	15/08/2012	13 & ff	N.P. Hessey	Calculate strip end points
F	17/02/2015	11 & ff	N.P. Hessey	Minor fixes, better notation, strip end calc.
G	19/02/2015	11 & ff	N.P. Hessey	Cut edges for curved ends and side-width differs from curved-end width

Contents

1 Introduction

Reference [1] gave a design for Endcap sensors with a stereo angle built in, which was optimised for a circumferential-type solution of building up an Endcap. The other option is to build an Endcap out of "petals" - wedge shaped support structures stretching all the way from the inner to outer radius of the Endcap strips region. There are at least three reasons to improve on the previous work:

- Petals need a different optimisation than circumferential designs: mainly because petals allow overlap of coverage in ϕ while the circumferential design allows overlap in r.
- The previous work did not specifiy the details of how the strips were layed out and rotated for stereo angle. Here we choose one particular solution.
- There was an approximation made in the previous document ("θ_g-left $= \theta_g$-right") which may cause difficulties some time in the future

2 Reference Frames

There are three reference frames used in this document, the "Sensor-frame", "Beam-frame", and "Strip-frame". Figure 2 illustrates these. The Sensor-frame has its origin at the centre of the original silicon wafer, and x-direction from thin-end to thick-end going radially outwards from the beam axis. The Beam-frame has its origin on the beam axis with x-axis parallel to the Xensor-frame x-axis. The Strip-frame has its origin at the focus f of all strips and x-axis along the bisector of the middle two strips of the sensor.

Transform from Beam-frame to Sensor-frame by translating $-R$ along the x axis:

$$\boldsymbol{p}_{Sensor} = \boldsymbol{p}_{Beam} + \begin{pmatrix} -R \\ 0 \end{pmatrix} \tag{1}$$

Transform from Sensor-frame to Strip-frame by rotating by the (negative) stereo-angle and translating R along the new x axis:

$$\boldsymbol{p}_{Strip} = \mathscr{R}(-\phi_s)\boldsymbol{p}_{Sensor} + \begin{pmatrix} R \\ 0 \end{pmatrix} \tag{2}$$

where $\mathscr{R}$ is the rotation matrix

$$\mathscr{R}(\phi) = \begin{pmatrix} \cos\phi & -\sin\phi \\ \sin\phi & \cos\phi \end{pmatrix} \tag{3}$$

3 Method Outline

In the current SCT Endcap, the wedges are symmetric about the x-axis, with strips separated by constant ϕ-pitch and pointing to a focus. If the sensor is not given a stereo-angle rotation, then this focus coincides with the beam-line: the strips are radial. In the current two-sided modules placed on discs, one side is radial, and the other given a 40 mrad stereo angle, giving so-called u or v strips. This allows the ϕ coordinate of a strip to be determined easily: ϕ is strip-pitch times strip number plus an offset.

Here I propose we keep this basic strip arrangement: constant ϕ-pitch, focussed on the beam-line if there is no stereo angle. If the sensor is rotated to give a stereo angle, the focus rotates away from the beam-line, but the focal distance R is kept the same, as in figure 2.

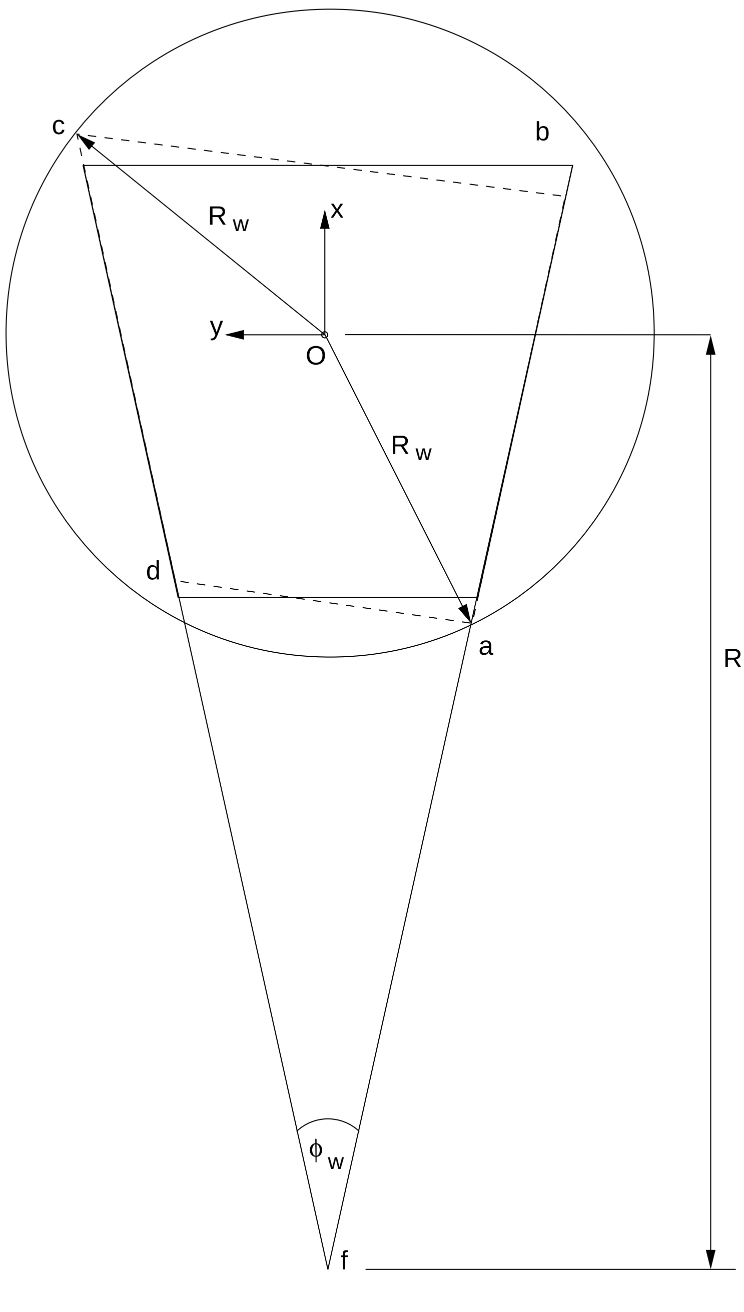

Figure 1: Figurative comparison of SCT-sensors (solid lines, zero stereo angle built in) to proposed wafers with stereo angle built in (dashed lines). The Sensor reference-frame is indicated: O is the centre of the original silicon wafer; R is the distance from O to the focal-point of the strips. The sensor covers an angle ϕ_w from the focal-point. Building in the stereo-angle shifts the sensor corners a, b, c, d so that when rotated by a stereo angle, a and d end up at one and the same radius from the beam-line, and b and c end up at another.

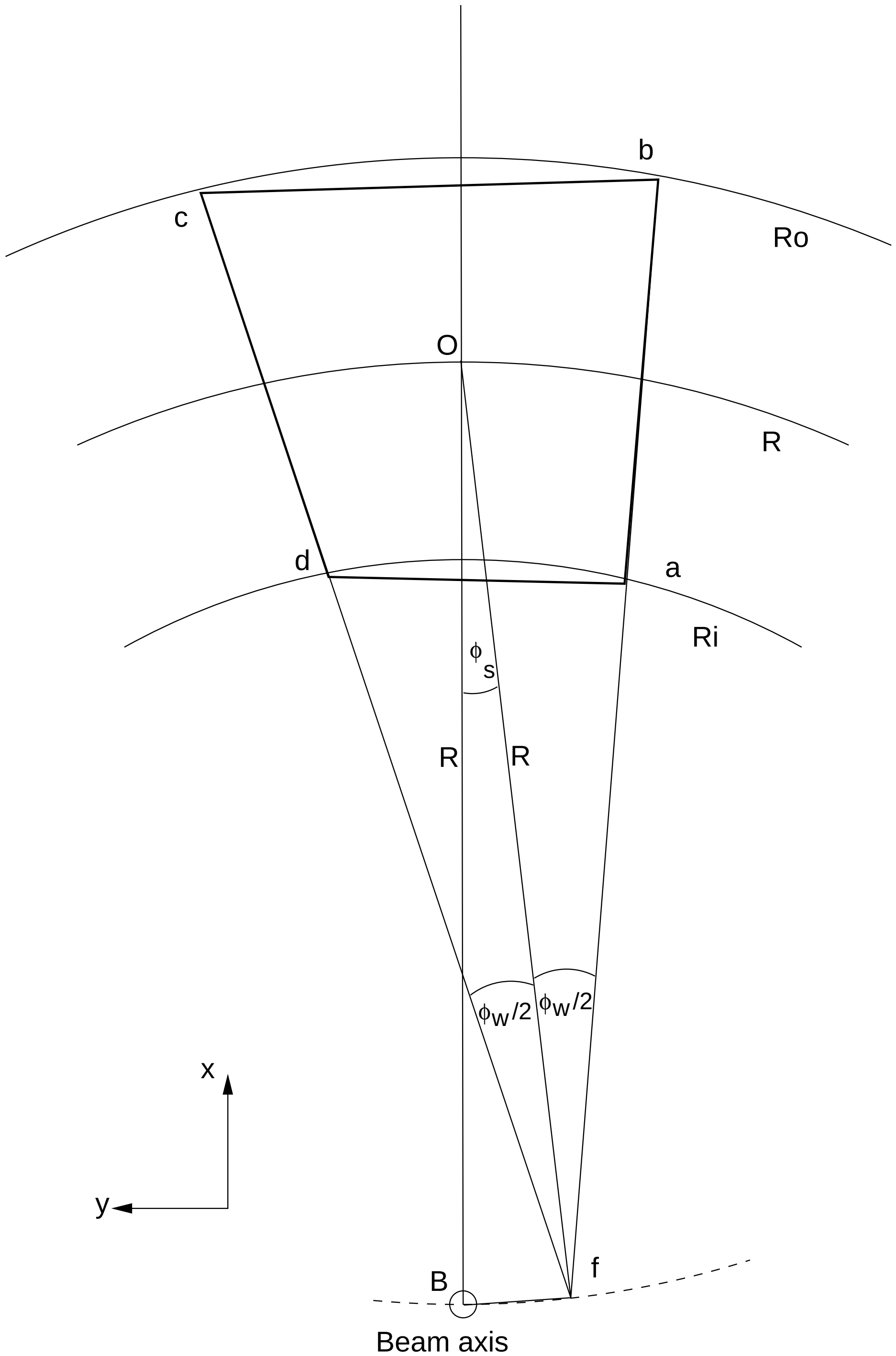

Figure 2: Beam reference frame with a sensor placed at the desired radius R from the beam-line and rotated by the stereo angle. The Sensor-frame is centred at O; the Strip-frame is centred at f.

It is not yet decided if the new Endcap will have radial plus u and v strips, or only u and v strips. The latter can halve the number of sensor designs and many other components, at a slight cost in performance (second-coordinate resolution deteriorates a factor $\sqrt{2}$, harder to implement fast track triggers). We keep all options open with the stereo angle ϕ_s as a (known) parameter.

We first calculate the dimensions of the *active area*: the region covered by strips. The actual sensors will have guard rings etc. in addition, and between sensors on a petal there will be gaps. This is calculated after the active area.

With petals, the ϕ-angle to be covered is known: typically we have 32 or 64 wedges in a complete ring, and we may want some overlap from one petal to the next, so the angle will be slightly larger than $2\pi/$(no. sensors in a ring). We denote the angle to be covered by a wafer as ϕ_w and treat it as a known quantity.

The sensors have to tessellate on a petal - i.e., cover the petal without clashes and with minimal gaps. Here things are very different to the SCT layout: the high-low staggering of modules on SCT discs allows rotation of wedges without interference to neighbouring modules. There are big advantages to building the stereo angle into the sensor: you avoid dead areas in the sensor, avoid some very short strips, and so on. We assume here that the stereo angle is built into the wafer. Figure 1 shows roughly what this means: the wafer is no longer symmetric about the x-axis, and neither the inner nor the outer corners are equidistant from the focal point – instead, they end up equidistant from the beam-axis. The main task is then to calculate the four corner positions of the active area.

In developing layouts, the radial region to be covered in the Endcap gets specified. Here I assume we start at the inner radius, and work outwards. Therefore I take the inner-radius R_i to be covered by a wafer as a known quantity; for the first ring, it is the same as the inner radius specified in the layout. The outer radius R_o of this ring has to be calculated; then for the next ring, R_i has to be chosen sufficiently larger than this R_o to allow for guard rings, clearances etc. All rings can then be built up sequentially.

We want to use as few silicon wafers as possible, so we would like to know what is the maximum strip length given R_i, ϕ_w, and ϕ_s. This depends on the allowed usable wafer dimension R_w (as well as the edge structures such as guard rings). We expect to use “six inch” wafers, which have an outer radius of 75 mm and usable radius of 70 mm. Allowing for guard rings etc. means that the usable radius for the active area is about 68 mm. R_w is left as a (known) parameter here.

The value of R_w puts a limit on R and R_o. The points a and c touch the circle of radius R_w centred at O. Since later we are going to add guard rings around the active area, we first have to reduce the radius from R_w to something smaller, to leave room for the guard rings. As a worst case (square sensors) we reduce R_w by $\sqrt{2} * g$ where g is the guard ring width. The first step in calculating the four corner points of a wafer involves calculating the maximum R possible. Of course, there may be reasons for using less than this, and so R could be adjusted downwards before calculating the corners if desired. Points a, d and c can then be calculated. Next the distance R_o is found, and finally point b.

Points $a, b, c,$ and d are first calculated in the Beam-frame. The final step transforms them to the Sensor-frame.

4 Sensor Positions in an Endcap

The choice of coordinate system given here has the convenience of allowing straightforward calculation of (ideal) sensor positions, for example in Geant simulations.

Imagine a sensor placed with O at the beamline, with its x-axis aligned with the Beam-frame x-axis. This sensor can be brought to its ideal position by a rotation about the beam-axis of ϕ_s, a translation along x of $\mathbf{R} = (R, 0)$, and a rotation about the beam axis of Φ to the desired ϕ position. In this transformation, a point $\mathbf{p}$ moves to $\mathbf{p}'$ given by:

$$\mathbf{p}' = \mathscr{R}(\Phi)(\mathbf{p} + \mathbf{R}) \tag{4}$$

5 Calculation of Active-area Shape

We assume $\phi_w, \phi_s, R_i,$ and R_w are known; also $\phi_s < \phi_w/2$. We want to derive R, R_o and the positions of a, b, c, d. We assume R_w has been adjusted downwards to leave space for the guard rings (see above).

Figure 2 shows the situation after positioning the sensor on a petal (with no ϕ rotation, $\Phi = 0$). The points B, O, and f are respectively the beam axis, the wafer centre, and the focus of the strips.

5.1 Finding the focal length R

We derive an implicit formula with R and all known quantities. This cannot be solved explicitly, which I think reflects the fact that figure 2 cannot be drawn until R is known.

Since there is an equal number of strips each side of the x-axis, the sensor active area subtends angles $\pm\phi_w/2$ from the focal point.

Note that ΔBfO is isosceles with apex ϕ_s, hence the other two angles are $\pi/2 - \phi_s/2$, and the length

$$(Bf) = 2R \sin \phi_s/2 \tag{5}$$

Hence the point f is

$$\begin{aligned} f_x &= (Bf) \sin \phi_s/2 \\ f_y &= -(Bf) \cos \phi_s/2 \end{aligned} \tag{6}$$

The line fa can be written

$$y = \alpha + \beta x \tag{7}$$

which crosses the x-axis at an angle $\pi - \phi_w/2 + \phi_s$ hence

$$\beta = -\tan(\phi_w/2 - \phi_s) \tag{8}$$

α is chosen to make the line pass through f, resulting in

$$\begin{aligned} \alpha &= -(Bf)(\beta \sin \phi_s/2 + \cos \phi_s/2) \\ &= kR \end{aligned} \tag{9}$$

where we introduce the shorthand

$$k = -2 \sin(\phi_s/2) [\beta \sin(\phi_s/2) + \cos(\phi_s/2)] \tag{10}$$

The sensor corner point a lies on this line and the circle of radius R_i, so

$$a_x^2 + a_y^2 = R_i^2 \tag{11}$$

Substituting for a_y from 7 gives a quadratic for a_x:

$$(1 + \beta^2) a_x^2 + 2\beta\alpha a_x + \alpha^2 - R_i^2 = 0 \tag{12}$$

the required solution takes the positive sign and simplifies to

$$a_x = \frac{-\beta kR + \sqrt{R_i^2(1+\beta^2) - k^2R^2}}{1+\beta^2} \tag{13}$$

Substituting back in 7 gives

$$a_y = \frac{kR + \beta\sqrt{R_i^2(1+\beta^2) - k^2R^2}}{1+\beta^2} \tag{14}$$

Denoting the angle OBa by $\phi_a(R)$ where we make the dependance on R explicit, we have

$$\begin{aligned} \phi_a(R) &= \arctan a_y/a_x \\ &= \arctan\left[\frac{kR + \beta\sqrt{R_i^2(1+\beta^2) - k^2R^2}}{-\beta kR + \sqrt{R_i^2(1+\beta^2) - k^2R^2}}\right] \end{aligned} \tag{15}$$

Using the cosine rule for triangle BOa gives

$$R_w^2 = R^2 + R_i^2 - 2RR_i \cos\phi_a(R) \tag{16}$$

This is an implicit equation for R, with only R unknown. The required root can easily be found by Newton's method. A good initial estimate for R is

$$R \approx R_i + R_w/\sqrt{2} \tag{17}$$

5.2 Coordinates of a

With R now known, the coordinates of a can be found from equations 13 and 14.

5.3 Coordinates of d

From figure 2, the angle Bfd is $\pi/2 - \phi_s/2 - \phi_w/2$. The sine rule applied to triangle Bdf gives the angle Bdf:

$$\begin{aligned} \angle Bdf &= \arcsin\left(\frac{(Bf)}{R_i}\sin\angle Bfd\right) \\ &= \arcsin\left[\frac{(Bf)}{R_i}\cos\left(\phi_s/2 + \phi_w/2\right)\right] \end{aligned} \tag{18}$$

Considering the triangles Bdf and OBf gives the polar-angle of d:

$$\phi_d = \phi_w/2 + \phi_s - \angle Bdf \tag{19}$$

so that d can be found from

$$\begin{aligned} d_x &= R_i \cos\phi_d \\ d_y &= R_i \sin\phi_d \end{aligned} \tag{20}$$

5.4 Coordinates of c

The coordinates of c are determined by the maximum usable radius R_w. First we find the length (fc) by applying the cosine rule to triangle fcO:

$$R_w^2 = R^2 + (fc)^2 - 2R(fc)\cos(\phi_w/2) \tag{21}$$

giving a quadratic in (fc) with required root

$$(fc) = R\cos(\phi_w/2) + \sqrt{R_w^2 - R^2\sin^2\phi_w/2} \tag{22}$$

Line (fc) makes an angle $\phi_s + \phi_w/2$ to the x-axis, allowing the x and y displacements of c from f to be found. Adding these to the coordinates of f gives c:

$$\begin{aligned} c_x &= f_x + (fc)\cos(\phi_s + \phi_w/2) \\ c_y &= f_y + (fc)\sin(\phi_s + \phi_w/2) \end{aligned} \tag{23}$$

5.5 Calculating the maximum outer radius covered, R_o

The corner c lies on R_o hence

$$R_o = \sqrt{c_x^2 + c_y^2} \tag{24}$$

5.6 Coordinates of b

Consider triangle bBf:

$$\angle Bfb = \pi/2 - \phi_s/2 + \phi_w/2 \tag{25}$$

The length (bf) can be found by applying the cosine rule

$$R_o^2 = (Bf)^2 + (fb)^2 - 2(Bf)(fb)\cos\angle Bfb \tag{26}$$

with required solution

$$(fb) = (Bf)\sin(\phi_s/2 - \phi_w/2) + \sqrt{R_o^2 - [1+\cos(\phi_s - \phi_w)](Bf)^2/2} \tag{27}$$

The line (fb) makes an angle $-(\phi_w/2 - \phi_s)$ with the x-axis, allowing the point b to be found:

$$\begin{aligned} b_x &= f_x + (fb)\cos(\phi_w/2 - \phi_s) \\ b_y &= f_y - (fb)\sin(\phi_w/2 - \phi_s) \end{aligned} \tag{28}$$

5.7 Active-area coordinates in the Sensor-frame

Finally we convert to the Sensor-frame using equation (1).

6 Sensor cut-edge coordinates

We now add a region of silicon around the active area, to hold guard rings and the extra space to the cut edge. The guard regions are curved at fixed distance g_h from the curved edges of the active area, and a possibly different width g_w at the straight sides. In practice these widths are made as small as is safe, but the sides need slightly more space to fit some bond pads on for the bias rail.

We first displace points a and b outwards by g_w in a direction perpendicular to ab to make new points a' and b', then do the same for c and d. We decrease R_i and increase R_o by g_h. Then we find the intersection of the lines $a'b'$ and $c'd'$ with these new arcs. These new points are the vertices of the cuts, for which we use capital letters A,B,C,D corresponding to a,b,c,d.

The vector of length g_w parallel to ab is

$$\mathbf{u} = g_w(\mathbf{b}-\mathbf{a})/||\mathbf{b}-\mathbf{a}|| \tag{29}$$

and this can be rotated by $-\pi/2$ to get a perpendicular vector $\mathbf{g}$

$$\begin{aligned} \mathbf{g}_{wx} &= \mathbf{u}_y \\ \mathbf{g}_{wy} &= -\mathbf{u}_x \end{aligned} \tag{30}$$

We displace a and b:

$$\begin{aligned} \mathbf{a}' &= \mathbf{a}+\mathbf{g} \\ \mathbf{b}' &= \mathbf{b}+\mathbf{g} \end{aligned} \tag{31}$$

We calculate the intersection of $a'b'$ with the displaced inner and outer arcs. Points $\mathbf{p}$ on the line $a'b'$ can be given in terms of a distance d from the point $\mathbf{a}'$ by

$$\mathbf{p} = \mathbf{a}' + d\mathbf{l} \tag{32}$$

where $\mathbf{l}$ is a unit vector along the line:

$$\mathbf{l} = (\mathbf{b}'-\mathbf{a}')/||\mathbf{b}'-\mathbf{a}'|| \tag{33}$$

Points on a circle radius r centred at $\mathbf{o}$ obey

$$||\mathbf{p}-\mathbf{o}||^2 = r^2 \tag{34}$$

Combining these and solving for d gives a quadratic formula with solutions (here b and c are coefficents – a is 1 – in the quadratic, not related to points $\mathbf{b}$ and $\mathbf{c}$):

$$\begin{aligned} d &= -b/2 \pm \sqrt{(b/2)^2-c} \\ b &\equiv 2\mathbf{l}\cdot(\mathbf{a}'-\mathbf{o}) \\ c &\equiv ||\mathbf{a}'-\mathbf{o}||^2-r^2 \end{aligned} \tag{35}$$

Similarly for cd. In the sensor coordinate system, $\mathbf{c} = (-R,0)$. We are interested in $r = R_i - g_h$ and $r = R_o + g_h$. Having found (the correct choice of) d, the points A,B,C,D can be found from adaptations of (32).

7 Hit-coordinates to strip calculation

In Geant simulation the digitisation step needs to work out which strip an energy deposit is in. Geant can give the deposit in the Sensor-frame in Cartesian coordinates as (x,y,z). To obtain the strip this lies in, we convert (x,y) to the Strip-frame in which all points along a strip have the same ϕ, then take the ϕ-coordinate of this point. Dividing by the angular strip-pitch and taking the nearest smaller integer gives an integer index s to the strip. This can be adjusted by a constant offset to give s in the required range, such as 0 to the number of strips less one.

Transforming to the Strip-frame is accomplished using equation (2). (Note: in GeoModel, the local reference frame is the beam-frame, and you need to combine this with (1)):

$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \mathscr{R}(-\phi_s) \begin{pmatrix} x \\ y \end{pmatrix} + \begin{pmatrix} R \\ 0 \end{pmatrix} \tag{36}$$

The azimuthal angle is

$$\phi' = \arctan(y'/x') \tag{37}$$

The strip-pitch ϕ_p, sensor-angle ϕ_w and number of strips N (assumed even) are related by

$$\phi_w = N\phi_p \tag{38}$$

and the strip index is

$$s = \mathrm{int}(\phi'/\phi_p) + N/2 \tag{39}$$

or

$$s = \mathrm{int}[N(\phi'/\phi_w + 0.5)] \tag{40}$$

s runs from 0 to $N-1$. $\mathrm{int}(x)$ returns the biggest integer not greater than x (floor function in C/C++).

8 Strip Positions

The position of each strip can be defined by its two endpoints and are useful e.g. to draw strips or calculates hit positions from a strip number. They can be calculated easily in the Strip Reference Frame (primeds frame in the following) where the azimuthal angle ϕ' is constant along the strip. Here we assume sensors have inner and outer edges and row-boundaries which are arcs of circles. The strips start and end close to these arcs, which are centred on the beam axis, not the strip focal point, and so require calculation of the intersection of a line and circle. The formula for this intersection is derived below in terms of the basic wafer parameters.

Typically one end of a strip implant is connected to a bias rail via a bias resistor, while the other end is not connected. These gaps g_b and g_{nb} are defined in fig. 3 and can be taken into account when calculating a radius r used below.

Figure (3) illustrates a strip at an angle ϕ' from the sensor centre line, intersecting the arc at radius $r = R_i$ from the beam axis at point p. The strip angle is related to its index s by

$$\phi' = (s - N/2 + 0.5)\phi_p \tag{41}$$

The radius of the intersection of this line with the arc at r is the length $r' = fp$. Consider $\triangle Bfp$. Length Bp is r; length Bf is given in equation (5); and the angle $\angle pfB$ is

$$\angle pfB = -\phi' + \angle BfO = \pi/2 - \phi_s/2 - \phi' \tag{42}$$

(see just before equation (5)). Note the sign of ϕ': in the figure, ϕ' is less than zero so must be *subtracted* from $\angle BfO$ to obtain $\angle pfB$. Using the cosine rule on $\triangle Bfp$ then gives

$$r^2 = r'^2 + (2R\sin\frac{\phi_s}{2})^2 - 2r'(2R\sin\frac{\phi_s}{2})\cos(\frac{\pi}{2} - \frac{\phi_s}{2} - \phi') \tag{43}$$

which can be rearranged into the standard quadratic form in r':

$$r'^2 + br' + c = 0 \tag{44}$$

with

$$b = -2(2R\sin\frac{\phi_s}{2})\sin(\frac{\phi_s}{2} + \phi') \tag{45}$$

and

$$c = (2R\sin\frac{\phi_s}{2})^2 - r^2 \tag{46}$$

with required solution

$$r' = (-b + \sqrt{b^2 - 4c})/2 \tag{47}$$

For a different radius arc to the one at R_i, just substitute its radius for R_i.

These are transformed to Cartesian coordinates:

$$x' = r'\cos\phi' \tag{48}$$

$$y' = r'\sin\phi' \tag{49}$$

and transformed to the sensor frame using the inverse of equation (2) (Note: in GeoModel frame, add $(R,0)$):

$$\begin{pmatrix} x \\ y \end{pmatrix} = \mathscr{R}(\phi_s)\left[\begin{pmatrix} x' \\ y' \end{pmatrix} + \begin{pmatrix} -R \\ 0 \end{pmatrix}\right] \tag{50}$$

Code has been written to draw the wafers in SVG-format. For this a final transformation of the coordinates is made:

$$x_{SVG} = kx + t \tag{51}$$

$$y_{SVG} = -ky + t \tag{52}$$

where k is a scale factor and t is an offset. The minus sign for y_{SVG} arises from SVG using y down the page instead of up.

9 Sensor area

The sensor area can be calculated starting with the area of the quadrilateral defined by the four corners, then adding the area obove the chord at the outer radius and subtracting the area above the chord at the inner radius.

The area of a quadrilateral defined by corners A, B, C, D with diagonals $\mathbf{p}$ and $\mathbf{q}$ is

$$A_Q = \frac{1}{2}|p_x q_y - q_x p_y| \tag{53}$$

with

$$\mathbf{p} = \mathbf{C} - \mathbf{A} \tag{54}$$

$$\mathbf{q} = \mathbf{D} - \mathbf{B} \tag{55}$$

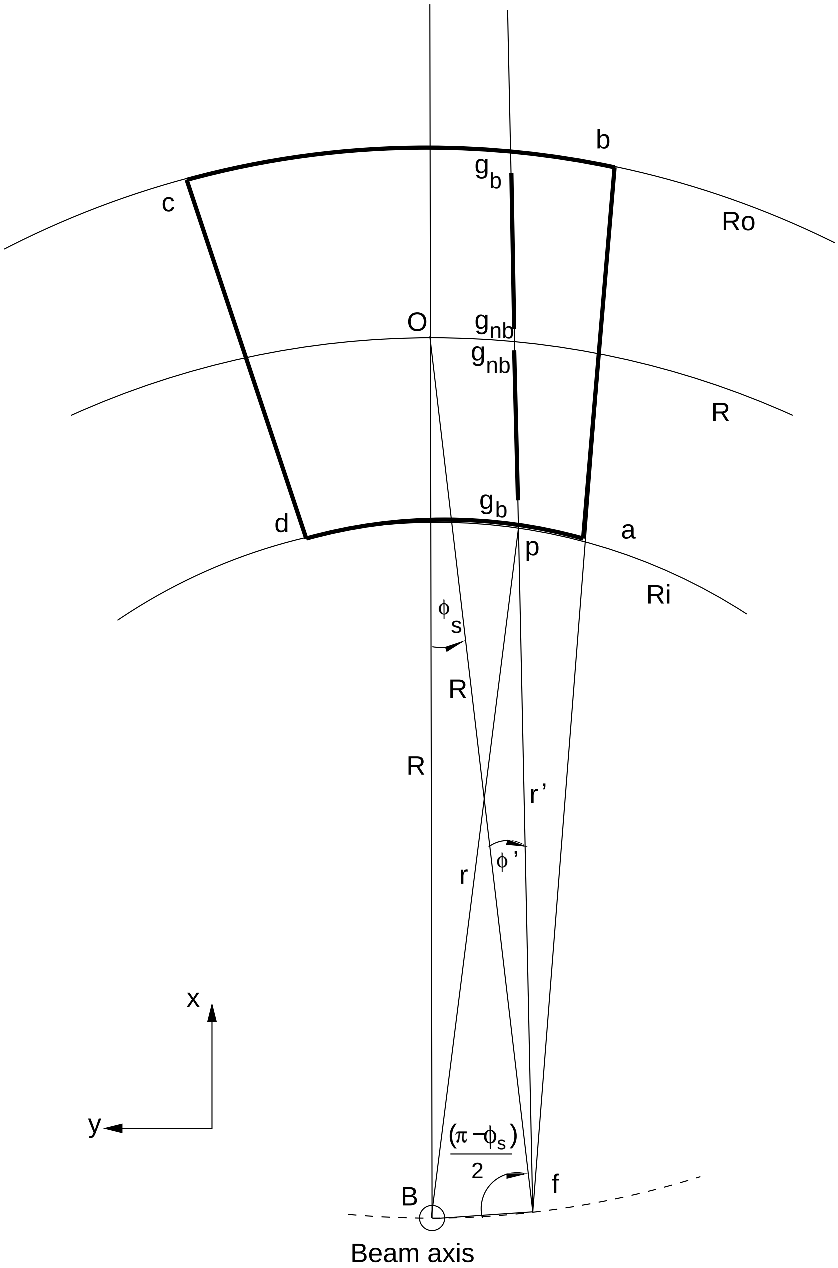

Figure 3: Geometry used for calculating strip positions.

The area bound by a chord of length c on a circle of radius r and subtending an angle θ at the circle centre is

$$A_C = \frac{r^2}{2}(\theta - \sin\theta) \tag{56}$$

with chord-length

$$c = \sqrt{(A_x + B_x)^2 + (A_y + B_y)^2} \tag{57}$$

and angle

$$\theta = 2\arcsin\frac{c}{2r} \tag{58}$$

10 Concluding remarks

A simple program called petalwafers has been written which implements these formulae. For a given ϕ_w, ϕ_s, Ri, Rw it calculates the four corners, R, and R_o. This can be used to build up a set of wafers to cover a petal. An alternative approach, probably better, would be to build up a spread-sheet using these formulae.

Having a method and a set of formulae allows flexibility as the layout evolves. It allows different layouts to be evaluated with accurate estimates of the number of sensors, chips etc. needed, and so can help optimise the layout. The geometry proposed here benefits from the same basic strip layout used in the SCT, but adapted to petals.

Precise scalable drawings of sensors in SVG format are created by the same program.

There is a lot more to making a layout than these few formulae: details such as gaps, overlaps, choosing the inner and outer radii of discs all have to be worked out. These formulae are just one ingredient.

References

[1] *A possible layout of silicon-strip sensors on end-cap discs with stereo angle*, N.P. Hessey, April 2007. Available at http://www.nikhef.nl/~r29/upgrade/stereodisc.pdf