	Petal Overlap Required for End-cap Hermeticity for 1 GeV/c tracks		
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Note

Petal Overlap Required for End-cap Hermeticity for 1 GeV/c tracks

This document presents some formulae for calculating how much overlap is needed on end-cap petals to ensure that all tracks above 1 GeV/c transverse-momentum must at least two sensors per disc.

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History of Changes

Rev. No.	*Date*	*Pages*	*Made by*	*Description of changes*
A	10/05/2012	All	N.P. Hessey	Original document
B	17/05/2012	New section	N.P. Hessey	Use distances following Lacasta
C	14/10/2012	New section	N.P. Hessey	Use in spreadsheet

1 Introduction

The method in overview is:

- Start with a standard mathematical paramterisation of a helix
- Change this to use the helix-axis coordinate (Z) as the parameter
- Transform from the standard coordinate frame to an ATLAS-like Cartesian frame
- Change from this to an ATLAS-like cylindrical coordinate reference frame
- Calculate $d\phi/dz$

This is then used to estimate the extra width needed in the sensors of a petal to give hermeticity.

2 Calculation

A standard helix parameterisation can be found for example in Wikipedia under "Helix". Cartesian coordinates (X,Y,Z) on the helix obey:

$$X = a\cos\Phi \tag{1}$$
$$Y = a\sin\Phi \tag{2}$$
$$Z = \frac{p}{2\pi}\Phi \tag{3}$$

Where a is the helix radius, p is the pitch (length along the Z-axis from $\Phi = 0$ to $\Phi = 2\pi$) and Φ is a parameter which is the angle of the line joining the point to the nearest point on the Z-axis. It is measured from the X-axis towards the Y-axis. The helix centre lies along the Z axis, and at the start ($\Phi = 0$) the helix is at $(a,0,0)$.
Re-parametrise this in terms of Z, noting from (3) that $\Phi = 2\pi Z/p$:

$$X = a\cos(2\pi Z/p) \tag{4}$$
$$Y = a\sin(2\pi Z/p) \tag{5}$$
$$Z = Z \tag{6}$$

Switch to an ATLAS-like Cartesian coordinate system (x,y,z) with z along the beam axis. Particles are created on this axis, initially directed with projection on the (x,y)-plane along the y-axis; the helix touches the z-axis at the completion of each pitch. In assuming this initial trajectory we simplify things but do not lose generality. This therefore requires a translation of $(-a, 0, 0)$ so the transformation equations are:

$$x = X - a \tag{7}$$
$$y = Y \tag{8}$$
$$z = Z \tag{9}$$

with inverse:

$$X = x + a \tag{10}$$
$$Y = y \tag{11}$$
$$Z = z \tag{12}$$

Eliminating (X, Y, Z) in (6) gives the helix in the ATLAS cartesian frame:

$$x = a(\cos(2\pi z/p) - 1) \tag{13}$$
$$y = a\sin(2\pi z/p) \tag{14}$$
$$z = z \tag{15}$$

The transformation equations from Cartesian to cylindrical are:

$$r = \sqrt{(x^2 + y^2)} \tag{16}$$
$$\phi = \arctan(y/x) \tag{17}$$
$$z = z \tag{18}$$

With inverse:

$$x = r\cos\phi \tag{19}$$
$$y = r\sin\phi \tag{20}$$
$$z = z \tag{21}$$

Converting to cylindrical coordinates gives the helix as:

$$r\cos\phi = a(\cos(2\pi z/p) - 1) \tag{22}$$
$$r\sin\phi = a\sin(2\pi z/p) \tag{23}$$
$$z = z \tag{24}$$

From which it is clear

$$\tan\phi = \frac{\sin(2\pi z/p)}{\cos(2\pi z/p) - 1} \tag{25}$$

Using the trigonometric identities

$$\sin 2\theta = 2\sin\theta\cos\theta \tag{26}$$
$$\cos 2\theta = \cos^2\theta - \sin^2\theta \tag{27}$$

this simplifies to

$$\tan\phi = -\cot(\pi z/p) \tag{28}$$

and using the identity

$$\tan\theta = -\cot(\theta - \pi/2) \tag{29}$$

this further simplifies to

$$\phi = \pi(\frac{z}{p} - \frac{1}{2}) \tag{30}$$

This very simple form gives the required derivative

$$\frac{d\phi}{dz} = \frac{\pi}{p} \tag{31}$$

Particles of charge q with initial direction θ to the z-axis moving in an axial magnetic field of magnitude B and with momentum magnitude P travel with pitch p and radius of curvature a given by:

$$p = \frac{2\pi P\cos\theta}{qB} \tag{32}$$

$$a = \frac{P\sin\theta}{qB} \tag{33}$$

(see standard texts or Google it) from which one sees

$$\frac{p}{a} = \frac{2\pi}{\tan\theta} \tag{34}$$

For the current problem, we want to calculate things at a given radius r_d on a disc positioned at $z = z_d$. For stiff tracks, tracks with $\tan\theta = r_d/z_d$ will reach this point. Hence

$$p = 2\pi a\frac{z_d}{r_d} \tag{35}$$

so that

$$\frac{d\phi}{dz} = \frac{1}{2a}\frac{r_d}{z_d} \tag{36}$$

Petals with no overlap are $2\pi/32$ rad wide. If two neighbouring petals in a disc are separated in z by $\triangle z > 0$ these would have a gap for low momentum tracks. Widening the active area of the petal can close this gap for particles above some minimum momentum. The two neighbouring edges of the neighbouring petals can be widened to close this gap. The extra width needed is approximately

$$\triangle w = \frac{1}{2}r_d\triangle z\frac{d\phi}{dz} \tag{37}$$

where we ignore the fact that the track also increases its radius in passing from the front petal to the rear one, and so reaches a wider part of the petal: so this is a conservative approximation. The factor of a half comes from the fact that both petals grow by this amount to make up the required total.
An additional factor of a half is needed where there are two sensors side by side on a petal: each sensor need only increase by a half of this w.

3 Alternative approach

Carlos Lacasta suggested another approach, avoiding switching to the ATLAS frame and restricting the calculation to use only distances between points, which are independant of the reference frame. Figure 1 shows a track hitting two successive petals on a disc, at points (x_1, y_1, z_1) and (x_2, y_2, z_2). These are separated in $r\phi$ by

$$r_d d\phi = v \sin\tau \tag{38}$$

where v is length of the hypotenuse of the red triangle and τ is the angle the track makes to the radial direction, which is also the angle at the upper corner of the red triangle.

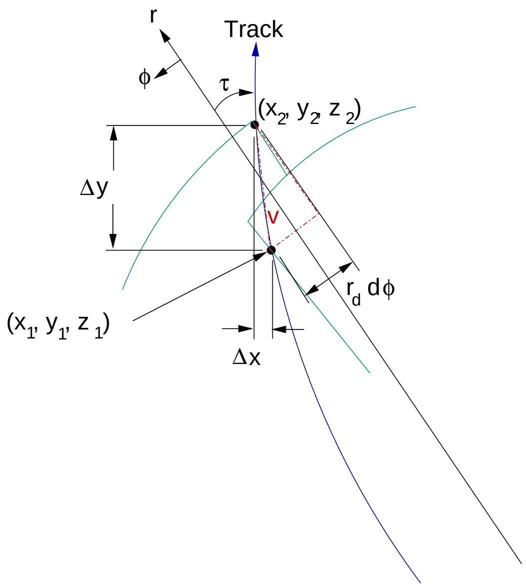

Figure 1: A track (blue) hitting the edges of two silicon sensors (cyan). The hypotenuse v can be calculated from Δx and Δy and used to obtain the required overlap $r_d d\phi$ using the red triangle. τ is the angle between the track and the radial direction.

Using equations (6) we can write x_i and y_i in terms of z_i. With the short-hand $c_i \equiv \cos(2\pi z_i/p)$ and similar for s_i gives

$$\begin{aligned}
v^2 &= (x_2 - x_1)^2 + (y_2 - y_1)^2 \\
&= a^2[(c_2 - c_1)^2 + (s_2 - s_1)^2] \\
&= 2a^2[1 - (s_1 s_2 + c_1 c_2)] \\
&= 2a^2[1 - \cos(2\pi\Delta z/p)] \\
&= 2a^2[1 - \cos^2(\pi\Delta z/p) + \sin^2(\pi\Delta z/p)] \\
&= 4a^2 \sin^2(\pi\Delta z/p)
\end{aligned} \tag{39}$$

where we have used several trigonomtric identities and arrive at

$$v = 2a \sin(\pi\Delta z/p) \tag{40}$$

In [1] it is shown (eq. (9) *loc cit*) that

$$\tan\tau = [(2a/r)^2 - 1]^{-1/2} \tag{41}$$

Using the trigonometric identity

$$\tan\theta = \pm\sin\theta/\sqrt{1-\sin^2\theta}, \tag{42}$$

solving for $\sin\tau$, and simplifying gives

$$\sin\tau = \frac{r}{2a} \tag{43}$$

Substituting for v and $\sin\tau$ applied at the disc-radius r_d into 38 gives

$$\begin{aligned} r_d d\phi &= 2a\sin(\pi\Delta z/p)\frac{r_d}{2a} \\ \Rightarrow d\phi &= \sin(\pi\Delta z/p) \\ &= \sin(\frac{1}{2a}\frac{r_d}{z_d}\Delta z) \end{aligned} \tag{44}$$

where the last step used (35). Given that $r_d < 2a$ and $\Delta z \ll z_d$ the sine-function can be dropped, recovering equation (36). Alternatively, this equation can be used directly (it gives the same results to at least 3 significant figures).

4 Comments for spread-sheet calculation

Take care to get factors of two right: the $r\phi$ overlap at each petal edge is accomplished half on one petal, and half on the other. But you need this overlap at both sides of a petal. So a petal needs to grow by the full $r\phi$ overlap.
On rings with two sensors per petal, each sensor contributes so each need only contribute half of what is wanted. These rings also have the sensors displaced in ϕ by half the separation between sensors, and by the guard-width. This reduces the need for extra width.
The program petalWafers needs the full dPhi as input in the geometry file.
These considerations were fixed in the spreadsheet Nov 14 2012. The end result was not much change, but individual corrections had significant effects.

5 Conclusions

A spread sheet has been created calculating the overlap needed for different places in the endcap. The most onerous disc is the one nearest the interaction point. This disc then sets the dimensions of the sensors to be modelled in Athena.

References

[1] *Calculation of Cylinder Radii in the Strip and Pixel Barrel Regions for the Inner Tracker Upgrade*, N.P. Hessey, Sep 2007, rev. C. Available at
`http://www.nikhef.nl/~r29/upgrade/barrelrad.pdf`