	Calculation of module overlaps in z-direction for barrel hermeticity		
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Note

Calculation of module overlaps in z-direction for barrel hermeticity

This note presents formulae for calculating the module positions along a barrel stave in order to give hermeticity in the r-z plane. The aim is to help with parametric layouts, which are easier to change as our understanding of the requirements develops.

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Distribution List:
Everybody

History of Changes

Rev. No.	*Date*	*Pages*	*Made by*	*Description of changes*
A	2/4/08	All	N Hessey	Original document
B	25/5/09	All	N Hessey	Minor wording changes
C	2/7/09	All	N Hessey	New more convenient method
D	30/9/11	All	N Hessey	Give explicitly 4 equations. Note constant pitch on each side.

1 Introduction

There are several options to explore for the ATLAS Inner Detector Upgrade layout. For z-overlaps, one can either allow gaps (not try to achieve hermeticity) with the benefit of simplicity of design, or choose a a scheme giving overlaps in z. One possibility for an hermetic layout is to have barrel modules arranged as in the current SCT, with successive modules along a row staggered in radius. This allows an overlap between neighbouring modules so tracks always pass through sensitive detector regions - i.e. giving hermetic coverage. The minimum amount of overlap needed for this can be calculated using a small number of assumptions and dimensions.

The calculations can be made in a spread sheet, allowing changes to be implemented easily. This has big advantages during the design phase, allowing different ideas to be tried.

One approach to making overlaps is described below.

2 Assumptions

We adopt the same overlap criteria as used for the SCT: the layout should guarantee hermeticity of each barrel layer for tracks with momentum above 1 GeV/c originating for the beam axis in a region $\pm 2\sigma_z$ about the central interaction point (IP), where σ_z is the collision distribution rms-width along the beam axis (z direction). In the z-direction, tracks are essentially straight lines in the r-z plane, and the momentum requirement is not relevant.

The current SCT has modules in an upper-lower "castellated" layout along a cylinder, with an even number of modules so that there is an overlap region at $z = 0$. The first module in the $z > 0$ direction is arbitrarily chosen to be an upper module. This is a general way of maintaining hermeticity in the z-direction: whether it is for SCT-type modules, or single sided modules along one side of a stave, or for pixel modules alternating on different sides of a stave.

Figure 1 shows this schematically for a stave with 8 modules and an exagerated gap between them.

Note that there are two types of transitions between sensors: as $|\eta|$ increases, the wafer traversed by tracks from the IP changes either from high to low (e.g. gap 1) or low to high (e.g. gap 2). The former needs wafers to overlap in z; the latter allows a gap in z while still ensuring hermeticity. Transitions from high-to-low I call inward and low-to-high I call outward. The critical tracks for inward transitions come from $2\sigma_z$ on the far side of the IP; and for outward transitions come from $2\sigma_z$ on the same side of the IP.

The overlap needed depends on the average radius r and radial separation Δr in the layer. If a single stave design is required to cover more than one layer, then the critical layer for inward transitions is the innermost layer to be covered; and for outward transitions is the outermost layer.

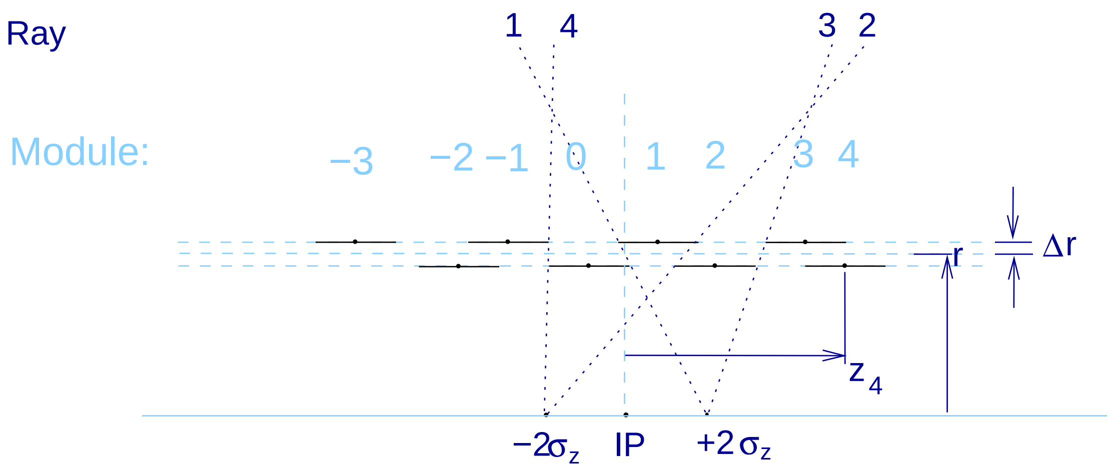

Figure 1: Module layout along a barrel, (r,z)-view.

First we derive the formula for a single layer of radius r; then extend this to cover a range of cylinders with radius r_I of the innermost and r_O of the outermost layer in the range.

3 Calculating the overlaps and module positions

We use the following notation: Figure 1 labels the modules from -3 to +4. The centre of module i is at z_i and these are the numbers to be calculated. The module-sensor active half-length is l. The origin is at the IP, positive z-direction along the beam line to the right, and r-direction vertically up; coordinates are given as (r,z).

There are many cases to consider: which radius r_I or r_O limits the position of the next sensor? Are we transiting from a lower to upper module or vice versa? Is $+2\sigma_z$ or $-2\sigma_z$ the limiting origin? Are we stepping to negative or positive z?

Assume as in the figure the first module is High. Then to place the next module in the negative-z direction, note the critical track is from $+2\sigma_z$. We draw a track from $+2\sigma_z$ through $(r+\Delta r, z_1-l)$ labelled Ray 1 in the figure. Then module -1 should be positioned to just touch this track at $r-\Delta r$.

From the triangle $(0,0)$ to $(0,+2\sigma_z)$ to $(r+\Delta r, z_1-l)$ and the smaller similar triangle $(0,z_{-1}+l)$ to $(0,2\sigma_z)$ to $(r-\Delta r, z_{-1}+l)$:

$$2\sigma_z-(z_{-1}+l)=\frac{r-\Delta r}{r+\Delta r}(2\sigma_z-(z_1-l)) \tag{1}$$

or

$$z_{-1}=\frac{r-\Delta r}{r+\Delta r}(-2\sigma_z+z_1-l)+2\sigma_z-l \tag{2}$$

In general, to go from the i^{th}-upper-module to the next lower-module at lower z

$$z_{i-1}=R(-2\sigma_z+z_i-l)+2\sigma_z-l \tag{3}$$

where

$$R=\frac{r-\Delta r}{r+\Delta r} \tag{4}$$

To go from the i^{th} upper module to the next lower module in the positive z direction, the critical tracks come from $-2\sigma_z$, Ray 2. Using triangles containing ray 2,

$$z_{i+1} = R(2\sigma_z + z_i + l) - 2\sigma_z + l \tag{5}$$

To go from lower modules to the next upper module at higher z using ray 3:

$$z_{i+1} = \frac{1}{R}(-2\sigma_z + z_i + l) + 2\sigma_z + l \tag{6}$$

To go from lower modules to the next upper module at lower z using ray 4:

$$z_{i-1} = \frac{1}{R}(2\sigma_z + z_i - l) - 2\sigma_z - l \tag{7}$$

These equations allow the distance to any neighbour module along z to be calculated.

Note that the distance between any two upper modules is constant; so is the distance between any lower two modules, but a different constant: Consider even i (lower modules), so both $i+1$ and $i-1$ are odd and label upper modules. Starting with eq. 5 then substituting z_i from eq. 6 with $i \to i-1$ gives

$$z_{i+1} - z_{i-1} = 2[2\sigma_z(R-1) + l(R+1)] \quad \text{(Upper modules).} \tag{8}$$

The right hand side is independent of i or any z and so is constant. Similarly for lower modules,

$$z_{i+1} - z_{i-1} = \frac{2}{R}[2\sigma_z(R-1) + l(R+1)] \quad \text{(Lower modules).} \tag{9}$$

If the same sensor arrangement is to be used on several barrels while giving hermetic cover on all, then r has to be replaced by the innermost barrel radius r_I for high to Low transitions, and the outermost r_O for low to high transitions; the R above then needs a subscript I or O.

4 Results

These formulae can be used in a spreadsheet to sequentially build up a table of z_n.